JOURNAL OF THE
CHUNGCHEONG MATHEMATICAL SOCIETY
Volume 22, No. 2, June 2009

THE GENERALIZED TRIANGULAR FUZZY SETS

Yong Sik Yun*, Sang Uk Ryu**, and Jin Won Park***

Abstract. For various fuzzy numbers, many operations have been calculated. We generalize about triangular fuzzy number and calculate four operations based on the Zadeh's extension principle, addition $A(+)B$, subtraction $A(-)B$, multiplication $A(\cdot)B$ and division $A(/)B$ for two generalized triangular fuzzy sets.

1. Introduction

The operations of two fuzzy numbers (A, μ_A) and (B, μ_B) defined in Definition 2.3 are based on the Zadeh's extension principle([4], [5], [6]). We consider four operations, addition $A(+)B$, subtraction $A(-)B$, multiplication $A(\cdot)B$ and division $A(/)B$ for two generalized triangular fuzzy sets A and B.

For two triangular fuzzy numbers, many results for various operations including the above four operations are known([1], [2]). About four operations, we introduce an example in Example 2.5.

In this paper, we generalize the triangular fuzzy number to generalized fuzzy set in Definition 3.1. It is a symmetric fuzzy set and may not have value 1. We study four operations for two generalized fuzzy sets in Theorem 3.2. The addition $A(+)B$ and subtraction $A(-)B$ becomes a generalized trapezoidal fuzzy set defined in Definition 2.5. But the multiplication $A(\cdot)B$ and division $A(/)B$ need not to be a generalized triangular fuzzy set or a generalized trapezoidal fuzzy set. And we give an example.

Received January 15, 2009; Accepted May 15, 2009.
2000 *Mathematics Subject Classifications*: Primary 47N30.
Key words and phrases: extension principle, generalized triangular fuzzy set.
Correspondence should be addressed to Yong Sik Yun, yunys@jejunu.ac.kr.
*Supported by the research grant of the Jeju National University in 2006.

2. Preliminaries

DEFINITION 2.1. *The set* $A_\alpha = \{x \in X \mid \mu_A(x) \geq \alpha\}$ *is said to be the* α*-cut of a fuzzy set* A.

The membership function of a fuzzy set A can be expressed in terms of the characteristic functions of its α-cuts according to the formula

$$\mu_A(x) = \sup_{\alpha\in(0,1]} \min(\alpha, \mu_{A_\alpha}(x)),$$

where

$$\mu_{A_\alpha}(x) = \begin{cases} 1, & x \in A_\alpha, \\ 0, & \text{otherwise.} \end{cases}$$

It is easily checked that the following properties hold

$$(A \cup B)_\alpha = A_\alpha \cup B_\alpha, \;\; (A \cap B)_\alpha = A_\alpha \cap B_\alpha.$$

DEFINITION 2.2. *A triangular fuzzy number is a fuzzy number* A *having membership function*

$$\mu_A(x) = \begin{cases} 0, & x < a_1, \;\; a_3 \leq x, \\ \frac{x-a_1}{a_2-a_1}, & a_1 \leq x < a_2, \\ \frac{a_3-x}{a_3-a_2}, & a_2 \leq x < a_3. \end{cases}$$

The above triangular fuzzy number is denoted by $A = (a_1, a_2, a_3)$.

DEFINITION 2.3. *The addition, subtraction, multiplication, and division of two fuzzy numbers are defined as*

1. Addition $A(+)B$ *:*

$$\mu_{A(+)B}(z) = \sup_{z=x+y} \min\{\mu_A(x), \mu_B(y)\}, \; x \in A, y \in B.$$

2. Subtraction $A(-)B$ *:*

$$\mu_{A(-)B}(z) = \sup_{z=x-y} \min\{\mu_A(x), \mu_B(y)\}, \; x \in A, y \in B.$$

3. Multiplication $A(\cdot)B$ *:*

$$\mu_{A(\cdot)B}(z) = \sup_{z=x\cdot y} \min\{\mu_A(x), \mu_B(y)\},\ x \in A, y \in B.$$

4. Division $A(/)B$:

$$\mu_{A(/)B}(z) = \sup_{z=x/y} \min\{\mu_A(x), \mu_B(y)\},\ x \in A, y \in B.$$

EXAMPLE 2.4. *For two triangular fuzzy numbers $A = (1, 2, 4)$ and $B = (2, 4, 5)$, we have*

1. Addition : $A(+)B = (3, 6, 9)$.

2. Subtraction : $A(-)B = (-4, -2, 2)$.

3. Multiplication :

$$\mu_{A(\cdot)B}(x) = \begin{cases} 0, & x < 2,\ \ 20 \le x, \\ \frac{-2+\sqrt{2x}}{2}, & 2 \le x < 8, \\ \frac{7-\sqrt{9+2x}}{2}, & 8 \le x < 20. \end{cases}$$

Note that $A(\cdot)B$ is not a triangular fuzzy number.

4. Division :

$$\mu_{A(/)B}(x) = \begin{cases} 0, & x < \frac{1}{5},\ \ 2 \le x, \\ \frac{5x-1}{x+1}, & \frac{1}{5} \le x < \frac{1}{2}, \\ \frac{-x+2}{x+1}, & \frac{1}{2} \le x < 2. \end{cases}$$

Note that $A(/)B$ is not a triangular fuzzy number.

DEFINITION 2.5. *A fuzzy set A having membership function*

$$\mu_A(x) = \begin{cases} 0, & x < a_1,\ \ a_4 \le x, \\ \frac{x-a_1}{a_2-a_1}, & a_1 \le x < a_2, \\ c, & a_2 \le x < a_3, \\ \frac{a_4-x}{a_4-a_3}, & a_3 \le x < a_4, \end{cases}$$

where $a_i \in \mathbb{R}, i = 1, 2, 3, 4$ and $0 < c \le 1$, is called a generalized trapezoidal fuzzy set.

The above generalized trapezoidal fuzzy set is denoted by $A = (a_1, a_2, c, a_3, a_4)$.

3. Generalized triangular fuzzy set

We generalize the triangular fuzzy number. A generalized triangular fuzzy set is symmetric and may not have value 1.

DEFINITION 3.1. *A generalized triangular fuzzy set is a symmetric fuzzy set A having membership function*

$$\mu_A(x) = \begin{cases} 0, & x < a_1, \ \ a_2 \le x, \\ \frac{2c(x-a_1)}{a_2-a_1}, & a_1 \le x < \frac{a_1+a_2}{2}, \\ \frac{-2c(x-a_2)}{a_2-a_1}, & \frac{a_1+a_2}{2} \le x < a_2, \end{cases}$$

where $a_1, a_2 \in \mathbb{R}$ and $0 < c \le 1$.

The above generalized triangular fuzzy set is denoted by $A = ((a_1, c, a_2))$.

THEOREM 3.2. *For two generalized triangular fuzzy sets $A = ((a_1, c_1, a_2))$ and $B = ((b_1, c_2, b_2))$, if $c_1 \le c_2$ and $\mu_B(x) \ge c_1$ in $[k_1, k_2]$, we have the followings.*

1. $A(+)B = (a_1 + b_1,\ \frac{1}{2}(a_1 + a_2) + k_1,\ c_1,\ \frac{1}{2}(a_1 + a_2) + k_2,\ a_2 + b_2)$, i.e., $A(+)B$ is a generalized trapezoidal fuzzy set.

2. $A(-)B = (a_1 - b_2,\ \frac{1}{2}(a_1 + a_2) - k_2,\ c_1,\ \frac{1}{2}(a_1 + a_2) - k_1,\ a_2 - b_1))$, i.e., $A(-)B$ is a generalized trapezoidal fuzzy set.

3. $A(\cdot)B$ is a fuzzy set on (a_1b_1, a_2b_2), but need not to be a generalized triangular fuzzy set or a generalized trapezoidal fuzzy set. The membership function of $A(\cdot)B$ is

$$\mu_{A(\cdot)B}(x) = \begin{cases} 0, & x < a_1b_1, \ \ a_2b_2 \le x, \\ \frac{1}{2pq}\left(-pb_1 - qa_1 + \sqrt{(pb_1 + qa_1)^2 - 4pq(a_1b_1 - x)}\right), & \\ & a_1b_1 \le x < a_1b_1 + \frac{1}{2}(a_1 + a_2)k_1, \\ \frac{1}{2}, \quad a_1b_1 + \frac{1}{2}(a_1 + a_2)k_1 \le x < a_1b_1 + \frac{1}{2}(a_1 + a_2)k_2, & \\ \frac{1}{2pq}\left(pb_2 + qa_2 - \sqrt{(pb_2 + qa_2)^2 - 4pq(a_2b_2 - x)}\right), & \\ & a_1b_1 + \frac{1}{2}(a_1 + a_2)k_2 \le x < a_2b_2, \end{cases}$$

where $p = \frac{a_2-a_1}{2c_1}$ and $q = \frac{b_2-b_1}{2c_2}$.

4. $A(/)B$ is a fuzzy set on $(\frac{a_1}{b_2}, \frac{a_2}{b_1})$, but need not to be a generalized triangular fuzzy set or a generalized trapezoidal fuzzy set. The membership function of $A(/)B$ is

$$\mu_{A(/)B}(x) = \begin{cases} 0, & x < \frac{a_1}{b_2}, \ \ \frac{a_2}{b_1} \le x, \\ \frac{2c_1c_2(b_2x - a_1)}{c_2(a_2-a_1)+c_1(b_2-b_1)x}, & \frac{a_1}{b_2} \le x < \frac{a_1+a_2}{2k_2}, \\ \frac{1}{2}, & \frac{a_1+a_2}{2k_2} \le x < \frac{a_1+a_2}{2k_1}, \\ \frac{-2c_1c_2(b_1x - a_2)}{c_2(a_2-a_1)+c_1(b_2-b_1)x}, & \frac{a_1+a_2}{2k_1} \le x < \frac{a_2}{b_1}. \end{cases}$$

Proof. Note that

$$\mu_A(x) = \begin{cases} 0, & x < a_1, \ \ a_2 \le x, \\ \frac{2c_1(x-a_1)}{a_2-a_1}, & a_1 \le x < \frac{a_1+a_2}{2}, \\ \frac{-2c_1(x-a_2)}{a_2-a_1}, & \frac{a_1+a_2}{2} \le x < a_2, \end{cases}$$

and

$$\mu_B(x) = \begin{cases} 0, & x < b_1, \ \ b_2 \le x, \\ \frac{2c_2(x-b_1)}{b_2-b_1}, & b_1 \le x < \frac{b_1+b_2}{2}, \\ \frac{-2c_2(x-b_2)}{b_2-b_1}, & \frac{b_1+b_2}{2} \le x < b_2. \end{cases}$$

We calculate exactly four operations using α-cuts. Let $A_\alpha = [a_1^{(\alpha)}, a_2^{(\alpha)}]$ and $B_\alpha = [b_1^{(\alpha)}, b_2^{(\alpha)}]$ are the α-cuts of A and B, respectively. Since $\alpha = \frac{2c_1(a_1^{(\alpha)}-a_1)}{a_2-a_1}$ and $\alpha = \frac{-2c_1(a_2^{(\alpha)}-a_2)}{a_2-a_1}$, we have

$$A_\alpha = [a_1^{(\alpha)}, a_2^{(\alpha)}] = \Big[\frac{(a_2-a_1)\alpha}{2c_1} + a_1, \frac{(a_2-a_1)\alpha}{-2c_1} + a_2\Big].$$

Similarly, we have

$$B_\alpha = [b_1^{(\alpha)}, b_2^{(\alpha)}] = \Big[\frac{(b_2-b_1)\alpha}{2c_2} + b_1, \frac{(b_2-b_1)\alpha}{-2c_2} + b_2\Big].$$

1. Addition : Since $c_1 \le c_2$ and $\mu_B(x) \ge c_1$ in $[k_1, k_2]$, $\mu_{A(+)B}(x) = c_1$ if $x \in [\frac{a_1+a_2}{2} + k_1, \frac{a_1+a_2}{2} + k_2]$. By the above facts,

$$\begin{aligned}A_\alpha(+)B_\alpha &= [a_1^{(\alpha)}+b_1^{(\alpha)}, a_2^{(\alpha)}+b_2^{(\alpha)}]\\ &= \Big[\frac{(a_2-a_1)\alpha}{2c_1}+a_1+\frac{(b_2-b_1)\alpha}{2c_2}+b_1,\\ &\qquad\qquad \frac{(a_2-a_1)\alpha}{-2c_1}+a_2+\frac{(b_2-b_1)\alpha}{-2c_2}+b_2\Big].\end{aligned}$$

If $x\in[a_1+b_1,\frac{a_1+a_2}{2}+k_1]$, then $\frac{(a_2-a_1)\alpha}{2c_1}+a_1+\frac{(b_2-b_1)\alpha}{2c_2}+b_1=x$. Thus calculating for α, $\alpha=\frac{2c_1c_2(x-a_1-b_1)}{c_2(a_2-a_1)+c_1(b_2-b_1)}$. Similarly, for $x\in[\frac{a_1+a_2}{2}+k_2,a_2+b_2]$, we have $\alpha=\frac{-2c_1c_2(x-a_2-b_2)}{c_2(a_2-a_1)+c_1(b_2-b_1)}$. Therefore

$$\mu_{A(+)B}(x)=\begin{cases}0, & x<a_1+b_1,\ \ a_2+b_2\le x,\\ \frac{2c_1c_2(x-a_1-b_1)}{c_2(a_2-a_1)+c_1(b_2-b_1)}, & a_1+b_1\le x<\frac{a_1+a_2}{2}+k_1,\\ \frac{1}{2}, & \frac{a_1+a_2}{2}+k_1\le x<\frac{a_1+a_2}{2}+k_2,\\ \frac{-2c_1c_2(x-a_2-b_2)}{c_2(a_2-a_1)+c_1(b_2-b_1)}, & \frac{a_1+a_2}{2}+k_2\le x<a_2+b_2.\end{cases}$$

Hence $A(+)B=(a_1+b_1,\ \frac{1}{2}(a_1+a_2)+k_1,\ c_1,\ \frac{1}{2}(a_1+a_2)+k_2,\ a_2+b_2)$, i.e., $A(+)B$ is a generalized trapezoidal fuzzy set.

2. Subtraction : Since $c_1\le c_2$ and $\mu_B(x)\ge c_1$ in $[k_1,k_2]$, $\mu_{A(-)B}(x)=c_1$ if $x\in[\frac{a_1+a_2}{2}-k_2,\frac{a_1+a_2}{2}-k_1]$. By the above facts,

$$\begin{aligned}A_\alpha(-)B_\alpha &= [a_1^{(\alpha)}-b_2^{(\alpha)}, a_2^{(\alpha)}-b_1^{(\alpha)}]\\ &= \Big[\frac{(a_2-a_1)\alpha}{2c_1}+a_1-\frac{(b_2-b_1)\alpha}{-2c_2}-b_2,\\ &\qquad\qquad \frac{(a_2-a_1)\alpha}{-2c_1}+a_2-\frac{(b_2-b_1)\alpha}{2c_2}-b_1\Big].\end{aligned}$$

If $x\in[a_1-b_2,\frac{a_1+a_2}{2}-k_2]$, then $\frac{(a_2-a_1)\alpha}{2c_1}+a_1-\frac{(b_2-b_1)\alpha}{-2c_2}-b_2=x$. Thus calculating for α, $\alpha=\frac{2c_1c_2(x-a_1+b_2)}{c_2(a_2-a_1)+c_1(b_2-b_1)}$. Similarly, for $x\in[\frac{a_1+a_2}{2}-k_1,a_2-b_1]$, we have $\alpha=\frac{-2c_1c_2(x-a_2+b_1)}{c_2(a_2-a_1)+c_1(b_2-b_1)}$. Therefore

$$\mu_{A(-)B}(x)=\begin{cases}0, & x<a_1-b_2,\ \ a_2-b_1\le x,\\ \frac{2c_1c_2(x-a_1+b_2)}{c_2(a_2-a_1)+c_1(b_2-b_1)}, & a_1-b_2\le x<\frac{a_1+a_2}{2}-k_2,\\ \frac{1}{2}, & \frac{a_1+a_2}{2}-k_2\le x<\frac{a_1+a_2}{2}-k_1,\\ \frac{-2c_1c_2(x-a_2+b_1)}{c_2(a_2-a_1)+c_1(b_2-b_1)}, & \frac{a_1+a_2}{2}-k_1\le x<a_2-b_1.\end{cases}$$

Hence $A(-)B = (a_1 - b_2,\ \frac{1}{2}(a_1+a_2) - k_2,\ c_1,\ \frac{1}{2}(a_1+a_2) - k_1,\ a_2 - b_1))$, i.e., $A(-)B$ is a generalized trapezoidal fuzzy set.

3. Multiplication : Since $c_1 \le c_2$ and $\mu_B(x) \ge c_1$ in $[k_1, k_2]$, $\mu_{A(\cdot)B}(x) = c_1$ if $x \in [\frac{a_1+a_2}{2}k_1, \frac{a_1+a_2}{2}k_2]$. By the above facts,

$$\begin{aligned} A_\alpha(\cdot)B_\alpha &= [a_1^{(\alpha)}b_1^{(\alpha)}, a_2^{(\alpha)}b_2^{(\alpha)}] \\ &= \Big[(\frac{(a_2-a_1)\alpha}{2c_1} + a_1)(\frac{(b_2-b_1)\alpha}{2c_2} + b_1), \\ &\qquad (\frac{(a_2-a_1)\alpha}{-2c_1} + a_2)(\frac{(b_2-b_1)\alpha}{-2c_2} + b_2)\ \Big]. \end{aligned}$$

If $x \in [a_1b_1, \frac{a_1+a_2}{2}k_1]$, then $(\frac{(a_2-a_1)\alpha}{2c_1} + a_1)(\frac{(b_2-b_1)\alpha}{2c_2} + b_1) = x$. Thus calculating for α,
$\alpha = \frac{1}{2pq}\big(-pb_1 - qa_1 + \sqrt{(pb_1+qa_1)^2 - 4pq(a_1b_1 - x)}\big)$, where $p = \frac{a_2-a_1}{2c_1}$ and $q = \frac{b_2-b_1}{2c_2}$. Similarly,
$\alpha = \frac{1}{2pq}\big(pb_2 + qa_2 - \sqrt{(pb_2+qa_2)^2 - 4pq(a_2b_2 - x)}\big)$
for $x \in [\frac{a_1+a_2}{2}k_2, a_2b_2]$. Therefore

$$\mu_{A(\cdot)B}(x) = \begin{cases} 0, & x < a_1b_1,\ \ a_2b_2 \le x, \\ \frac{1}{2pq}\big(-pb_1 - qa_1 + \sqrt{(pb_1+qa_1)^2 - 4pq(a_1b_1 - x)}\big), & \\ \qquad a_1b_1 \le x < a_1b_1 + \frac{1}{2}(a_1+a_2)k_1, & \\ \frac{1}{2}, \quad a_1b_1 + \frac{1}{2}(a_1+a_2)k_1 \le x < a_1b_1 + \frac{1}{2}(a_1+a_2)k_2, & \\ \frac{1}{2pq}\big(pb_2 + qa_2 - \sqrt{(pb_2+qa_2)^2 - 4pq(a_2b_2 - x)}\big), & \\ \qquad a_1b_1 + \frac{1}{2}(a_1+a_2)k_2 \le x < a_2b_2, & \end{cases}$$

where $p = \frac{a_2-a_1}{2c_1}$ and $q = \frac{b_2-b_1}{2c_2}$.
Hence $A(\cdot)B$ is a fuzzy set on (a_1b_1, a_2b_2), but need not to be a generalized triangular fuzzy set or a generalized trapezoidal fuzzy set.

4. Division : Since $c_1 \le c_2$ and $\mu_B(x) \ge c_1$ in $[k_1, k_2]$, $\mu_{A(/)B}(x) = c_1$ if $x \in [\frac{a_1+a_2}{2k_2}, \frac{a_1+a_2}{2k_1}]$. By the above facts,

$$\begin{aligned} A_\alpha(/)B_\alpha &= [a_1^{(\alpha)}/b_2^{(\alpha)}, a_2^{(\alpha)}/b_1^{(\alpha)}] \\ &= \Big[\frac{\frac{(a_2-a_1)\alpha}{2c_1} + a_1}{\frac{(b_2-b_1)\alpha}{-2c_2} + b_2}, \frac{\frac{(a_2-a_1)\alpha}{-2c_1} + a_2}{\frac{(b_2-b_1)\alpha}{2c_2} + b_1}\ \Big]. \end{aligned}$$

If $x \in [\frac{a_1}{b_2}, \frac{a_1+a_2}{2k_2}]$, then $(\frac{(a_2-a_1)\alpha}{2c_1}+a_1)/(\frac{(b_2-b_1)\alpha}{-2c_2}+b_2) = x$. Thus calculating for α, $\alpha = \frac{2c_1c_2(b_2x-a_1)}{c_2(a_2-a_1)+c_1(b_2-b_1)x}$. Similarly, for $x \in [\frac{a_1+a_2}{2k_1}, \frac{a_2}{b_1}]$, we have $\alpha = \frac{-2c_1c_2(b_1x-a_2)}{c_2(a_2-a_1)+c_1(b_2-b_1)x}$. Therefore

$$\mu_{A(/)B}(x) = \begin{cases} 0, & x < \frac{a_1}{b_2}, \ \frac{a_2}{b_1} \le x, \\ \frac{2c_1c_2(b_2x-a_1)}{c_2(a_2-a_1)+c_1(b_2-b_1)x}, & \frac{a_1}{b_2} \le x < \frac{a_1+a_2}{2k_2}, \\ \frac{1}{2}, & \frac{a_1+a_2}{2k_2} \le x < \frac{a_1+a_2}{2k_1}, \\ \frac{-2c_1c_2(b_1x-a_2)}{c_2(a_2-a_1)+c_1(b_2-b_1)x}, & \frac{a_1+a_2}{2k_1} \le x < \frac{a_2}{b_1}. \end{cases}$$

Hence $A(/)B$ is a fuzzy set on $(\frac{a_1}{b_2}, \frac{a_2}{b_1})$, but need not to be a generalized triangular fuzzy set or a generalized trapezoidal fuzzy set. □

EXAMPLE 3.3. *Let* $A = ((2, \frac{1}{2}, 8))$ *and* $B = ((1, \frac{4}{5}, 5))$ *be generalized triangular fuzzy sets, i.e.,*

$$\mu_A(x) = \begin{cases} 0, & x < 2, \ 8 \le x, \\ \frac{1}{6}(x-2), & 2 \le x < 5, \\ -\frac{1}{6}(x-8), & 5 \le x < 8, \end{cases}$$

and

$$\mu_B(x) = \begin{cases} 0, & x < 1, \ 5 \le x, \\ \frac{2}{5}(x-1), & 1 \le x < 3, \\ -\frac{2}{5}(x-5), & 3 \le x < 5. \end{cases}$$

Let A_α *and* B_α *be the* α*-cuts of* A *and* B*, respectively. Let* $A_\alpha = [a_1^{(\alpha)}, a_2^{(\alpha)}]$ *and* $B_\alpha = [b_1^{(\alpha)}, b_2^{(\alpha)}]$*. Since* $\alpha = \frac{1}{6}(a_1^{(\alpha)} - 2)$ *and* $\alpha = -\frac{1}{6}(a_2^{(\alpha)} - 8)$*,* $A_\alpha = [a_1^{(\alpha)}, a_2^{(\alpha)}] = [6\alpha+2, -6\alpha+8]$*. Since* $\alpha = \frac{2}{5}(b_1^{(\alpha)} - 1)$ *and* $\alpha = -\frac{2}{5}(b_2^{(\alpha)} - 5)$*, we have* $B_\alpha = [b_1^{(\alpha)}, b_2^{(\alpha)}] = [\frac{5}{2}\alpha + 1, -\frac{5}{2}\alpha + 5]$*. Then we have the followings.*

1. Addition :

$$\mu_{A(+)B}(x) = \begin{cases} 0, & x < 3, \ 13 \le x, \\ \frac{2}{17}(x-3), & 3 \le x < \frac{29}{4}, \\ \frac{1}{2}, & \frac{29}{4} \le x < \frac{35}{4}, \\ \frac{-2}{17}(x-13), & \frac{35}{4} \le x < 13, \end{cases}$$

i.e., $A(+)B = (3, \frac{29}{4}, \frac{1}{2}, \frac{35}{4}, 13)$.

2. Subtraction :

$$\mu_{A(-)B}(x) = \begin{cases} 0, & x < -3, \ \ 7 \le x, \\ \frac{2}{17}(x+3), & -3 \le x < \frac{5}{4}, \\ \frac{1}{2}, & \frac{5}{4} \le x < \frac{11}{4}, \\ \frac{-2}{17}(x-7), & \frac{11}{4} \le x < 7, \end{cases}$$

i.e., $A(-)B = (-3, \frac{5}{4}, \frac{1}{2}, \frac{11}{4}, 7)$.

3. Multiplication :

$$\mu_{A(\cdot)B}(x) = \begin{cases} 0, & x < 2, \ \ 40 \le x, \\ \frac{1}{30}(-11 + \sqrt{121 - 60(2-x)}), & 2 \le x < \frac{45}{4}, \\ \frac{1}{2}, & \frac{45}{4} \le x < \frac{75}{4}, \\ \frac{1}{30}(50 - \sqrt{2500 - 60(40-x)}), & \frac{75}{4} \le x < 40, \end{cases}$$

Thus $A(\cdot)B$ *is a fuzzy set on* $(2, 40)$*, but not a generalized triangular fuzzy set or a generalized trapezoidal fuzzy set.*

4. Division :

$$\mu_{A(/)B}(x) = \begin{cases} 0, & x < \frac{2}{5}, \ \ 8 \le x, \\ \frac{10x-4}{5x+12}, & \frac{2}{5} \le x < \frac{4}{3}, \\ \frac{1}{2}, & \frac{4}{3} \le x < \frac{20}{9}, \\ \frac{-2(x-8)}{5x+12}, & \frac{20}{9} \le x < 8, \end{cases}$$

Thus $A(/)B$ *is a fuzzy set on* $(\frac{2}{5}, 8)$*, but not a generalized triangular fuzzy set or a generalized trapezoidal fuzzy set.*

References

[1] Y. S. Yun, J. C. Song and J. W. Park, *Normal fuzzy probability for quadratic fuzzy number*, Journal of fuzzy logic and intelligent systems **15** (2005), no. 3, 277–281.

[2] Y. S. Yun, J. C. Song and S. U. Ryu, *On the exponential fuzzy probability*, Communications of the Korean Mathematical Society **21** (2006), no. 2, 385–395.

[3] L. A. Zadeh, *Probability measures of fuzzy events* J. Math. Anal. Appl. **23** (1968), 421–427.

[4] L. A. Zadeh, *The concept of a linguistic variable and its application to approximate reasoning -I*, Information Sciences **8** (1975), 199–249.

[5] L. A. Zadeh, *The concept of a linguistic variable and its application to approximate reasoning -II*, Information Sciences **8** (1975), 301–357.

[6] L. A. Zadeh, *The concept of a linguistic variable and its application to approximate reasoning -III*, Information Sciences **9** (1975), 43–80.

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Department of Mathematics
Jeju National University
Jeju 690-756, Republic of Korea
E-mail: yunys@jejunu.ac.kr

**
Department of Mathematics
Jeju National University
Jeju 690-756, Republic of Korea
E-mail: ryusu81@jejunu.ac.kr

***
Department of Mathematics Education
Jeju National University
Jeju 690-756, Republic of Korea
E-mail: jinwon@jejunu.ac.kr