논 문 초 록 집

2013년도 연 구 발 표 회

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충 청 수 학 회

발표장 및 행사장 안내

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제 2 연구발표장 ..... 사범대학 30117호실

제 3 연구발표장 ..... 사범대학 30118호실

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2013년도 연구발표회

한남대학교

May 11, 2013 (Saturday)

(* : 발표자)

초청강연

IN-1 조현숙 (ETRI, 한국전자통신연구원, 사이버보안연구단장) : 정보보안의 글로벌 연구 동향

환경 변화에 따른 기존의 통신상의 정보보호에서 생활 속의 정보보안으로 패러다임 변화에 대응하여 보안 기술도 다각도의 접근을 요구하고 있다. 본 강연에서는 IT 기술 및 보안 기술 트렌드를 살펴보고 연구 방향을 논하고자 함.

초청강연

IN-1 김동수 (NIMS, 국가수리과학연구소장) : NIMS의 역할과 연구분야

본 강연에서는 국가수리과학연구소(NIMS)의 역할과 연구분야를 소개한다.

1장 'NIMS, 무엇을 위한 연구기관인가'에서는 NIMS의 구성, 임무, 계획에 관해 소개하고,

2장 'NIMS, 연구분야'에서는 NIMS가 강조하는 연구 주제들을 소개하고,

3장 'NIMS, 수학계의 리서치허브(Research Hub)'에서는 수학원리응용센터를 소개하고,

마지막으로 '수학연구의 경향'에서는 최근 국내외에서 중요한 연구주제로 부각되는

빅데이터 분야에서 수학적 분석의 중요성과 수학자의 역할을 소개하며 최근 수학연구 경향을 논한다.

제1 발표장(사범대학 30111호실)

1-1 임성근(목원대학교) : A few classes of infinite series identities

We see that a lot of infinite series identities can be obtained from a modular transformation formula which comes from the generalized Eisenstein series. In this talk, we find a few classes of new infinite series using a more complicated modular transformation formula.

1-2 임정욱 (경북대학교) : Integral domains in which every t-locally principal ideal is t-invertible

Bazzoni conjectured that if D is a Prüfer domain in which every nonzero locally principal ideal is invertible, then D is of finite character. This conjecture was solved in the affirmative by several authors. Later, Anderson and Zafrullah called a domain D an *LPI domain* if every nonzero locally principal ideal of D is invertible. Halter-Koch and Zafrullah also studied the PvMD analogue of Bazzoni's conjecture that a PvMD D is of finite t-character if and only if every t-locally principal t-ideal of D is t-invertible. In this talk, we investigate some properties of integral domains in which every nonzero t-locally principal ideal is t-invertible.

This is a joint work with G.W. Chang and H. Kim.

1-3 오세권(충남대학교), 박수진(*) (충남대학교) : Poisson brackets determined by Jacobians

Fix $n-2$ elements $h_1, \cdots, h_{n-2}$ of the quotient field B of the polynomial algebra $\mathbb{C}[x_1, x_2, \cdots, x_n]$. It is proved that B is a Poisson algebra with Poisson bracket defined by

$$\{f, g\} = \det(\mathrm{Jac}(f, g, h_1, \cdots, h_{n-2}))$$

for any $f, g \in B$, where det(Jac) is the determinant of a Jacobian matrix.

1-4 최호원(*)(고려대학교), 이기영 (고려대학교) : Certain invariants of CW-complexes

For any CW complex X, the group $\varepsilon(X)$ of self homotopy equivalences of X is related with the ordinary homotopy group $\pi_i(X)$ for $i \geq 0$. We develop relations between them and introduce certain invariant number which is related with the groups of self homotopy equivalences. The number is called the self number of X and denoted by $N_{\varepsilon(X)}$. We introduce the set $Aut_{\sharp k}(X)$. A map $f \in Aut_{\sharp k}(X)$ if and only if f is a self map from X to itself and induces the homomorphism $\pi_i(f)$ which is an isomorphism from $\pi_i(X)$ to itself for $i = 0, 1, \cdots, k$. The self number $N_{\varepsilon(X)}$ is the minimum number k such that $\varepsilon(X) = Aut_{\sharp k}(X)$. For the number, we prove the following three theorems.

Theorem 1. Let X and Y be CW complexes. If X and Y have same homotopy type then $N_{\varepsilon(X)} = N_{\varepsilon(Y)}$.

Theorem 2. Let X and Y be CW complexes. If $N_{\varepsilon(X)} = m$ and $N_{\varepsilon(Y)} = n$ then $N_{\varepsilon(X\times Y)} \geq \max\{m, n\}$.

We define

$$\{X, Y\} := [X, Y] \cong [S^n X, S^n Y],$$

where $S^n X$ is the n-fold suspension of X for $n = 0, 1, \cdots$.

Theorem 3. If $\{X, X\}$, $\{S^i, X\}$ and $N_{\varepsilon(X)} = n$ where $i = 0, 1, 2..., n$, then $N_{\varepsilon(s^k X)} = n + k$ for $k = 1, 2, \ldots$.

1-5 윤연수 (한남대학교) : On some generalizations of C_k^f-spaces

For a map $f : A \to X$, a space X is called a C_k^f-*space* if the inclusion $e_k^X : P^k(\Omega X) \to X$ is f-cyclic. The concepts of C_k^f-spaces are closely related by Gottlieb sets and LS-category as follows; It is shown that a space X is a C_k^f-space if and only if $G^f(Z, X) = [Z, X]$ for any space Z with $cat\ Z \leq k$.

In this talk, we study some generalizations of C_k^f-spaces taking more weaker conditions than those to be C_k^f-spaces.

1) In general, $G^f(Z, X) \subset WG^f(Z, X)$.

What is the space X satisfying $WG^f(Z, X) = [Z, X]$ for any space Z with $cat\ Z \leq k$?

2) In general, the concept of $mapcat\ (Z, X) \leq k$ is a weaker than that of $cat\ Z \leq k$.

What is the space X satisfying $G^f(Z, X) = [Z, X]$ for any space Z with $mapcat\ (Z, X) \leq k$?

1-6 이진호(*)(고려대학교), 이기영 (고려대학교) : Cocyclic morphisms with respect to a map

We introduce the notion of cocylic morphisms with respect to a map. It is defined in the category of pairs proposed by P. Hilton. The cocylic morphism with respect to a map is the dual concept of a cyclic morphism of a map and a generalization of a coyclic morphism in the category of pairs. We study its basic properties such as preservation of cocyclicity by morphisms and give some conditions of the set of all cocyclic morphisms with respect to a map to be a group. We show that there is a relation between cocyclic morphism with respect to a map and the A-category length or A-cone length. Finally we introduce a generalized dual G-sequence and study the condition of the sequence to be exact.

1-7 오형석(*)(고려대학교), 이기영 (고려대학교) : Self homotopy equivalences on Moore spaces which induce the identity on cohomotopy groups

In this paper, we introduce $\varepsilon_k^{\#}(X)$ as a dual concept of $\varepsilon_\sharp(X)$ and study properties of the concept. $\varepsilon_k^{\#}(X)$ is the set of self-homotopy equivalences which induce the identity from l-th cohomotopy group to itself for all $l \geq k$. By using several exact sequences in homotopy theory, we obtain concrete computations of $\varepsilon_k^{\#}(M(\mathbb{Z}_q, n))$ for $k = n-1, n, n+1$, where $M(\mathbb{Z}_q, n)$ is a Moore space. By Araki and Toda, $[S^{n+i-3}M_q, S^n]$, $[S^{n+i}, S^n M_q]$ and $[M(\mathbb{Z}_q, n)), M(\mathbb{Z}_q, n))]$ were computed, where M_q is a Moore space $M(\mathbb{Z}_q, 1)$. To compute $\varepsilon_k^{\#}(M(\mathbb{Z}_q, n))$ for $k = n-1, n, n+1$, we use a table of computations of $[S^{n+i-3}M_q, S^n]$, $[S^{n+i}, S^n M_q]$ and $[M(\mathbb{Z}_q, n)), M(\mathbb{Z}_q, n))]$.

1-8 고두원(충북대학교) : Restriction operators acting on radial functions on vector spaces over finite fields

In this talk, we stduy $L^p - L^r$ restriction estimates for algebraic varieties in the case when restriction operators act on radial functions in the finite field setting. We show that if the varieties lie in odd dimensional vector spaces over finite fields, then the conjectured restriction estimates are possible for all radial test functions. In addition, it is proved that if the varieties in even dimensions have few intersection points with the sphere of zero radius, then the same conclusion as in odd dimensional case can be also obtained.

1-9 한성휴(한국기술교육대학교) : A method for constructing self-dual codes over $\mathbb{Z}_{2^m}$

Hahn [Hahn] established three differential equations according to $\mathcal{P}(q)$, $\mathcal{E}(q)$, and $\mathcal{Q}(q)$, which allows us to obtain the formulas for

$$\sum_{l+m+n=N} \tilde{\sigma}_1(l)\tilde{\sigma}_1(m)\tilde{\sigma}_1(n), \qquad \sum_{l+m+n=N} \widehat{\sigma}_1(l)\widehat{\sigma}_1(m)\tilde{\sigma}_1(n),$$

etc. Finally, by using the above equations we derive the algebraic curves.

[Hahn] H. Hahn, *Convolution sums of some functions on divisors*, Rocky Mountain J. Math. **37** (2007), 1593–1622.

1-10 김학만(충남대학교), 손은영 (*)(충남대학교) : Generalized Hyers–Ulam Stability of Cauchy–Jensen type functional equation in quasi-β-Banach spaces

In this talk, we prove the generalized Hyers–Ulam stability of the following generalized Cauchy–Jensen type functional equation

$$f(\frac{x+y}{n}+z)+f(\frac{y+z}{n}+x)+f(\frac{z+x}{n}+y)=\frac{n+2}{n}[f(x)+f(y)+f(z)]$$

in (β,p)–Banach spaces for any fixed nonzero integer n.

제 2발표장 (사범대학 30117호실)

2-1 이경자(한남대학교) : Intertwining Numbers; The n-rowed partitions with $d = 1, 2$ and the four-rowed partitions

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In modular representation theory, the intertwining number, i.e., the dimension of the $\mathbb{Z}/(p)$-vector space $Hom_{\overline{A}_r}(\overline{K}_\lambda, \overline{K}_\mu)$ for any partition λ and μ of r, is one of the fairly old problems. The cases of partitions of length two(1990) and three(1995, 1996) have been solved by K.Akin, D.A.Buchsbaum, and D.Flores. Also, H.J.Ko and K.J.Lee proved this problem for any n-rowed partition $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_n)$ and $\mu = (\lambda_1 + 1, \lambda_2, \cdots, \lambda_n - 1)$ in 2007. In this paper, we compute the intertwining number of $\overline{K}_\lambda$ and $\overline{K}_\mu$ for the cases of partition $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_n)$ and $\mu = (\lambda_1 + 2, \lambda_2, \cdots, \lambda_n - 2)$, and we determine the intertwining number of Weyl modules for the four-rowed partitions.

2-2 유충현(한남대학교) : 수학 개념의 인식론적 고찰

수학 개념은 어떻게 해서 가능한가? 칸트의 인식론에 따르면, 수학 개념은 현상을 시간과 공간이라는 순수 직관을 가지고 능동적으로 수용하고 그것을 수학적인 양과 질의 범주로 인식함으로 가능하다. 인간의 마음은 세계와 접하여 시간과 공간 직관으로 현상을 수용하고 이를 범주에 의해 수학적 개념으로 구성한다. 시간 직관으로 수용된 현상에 대한 오성의 종합으로부터 수 개념이 구성되며, 공간 직관에 수용된 현상에 대한 오성의 종합으로부터 도형 개념이 구성된다. 세계 인식과 수학은 불가분의 관계에 있으므로, 세계 인식 능력, 곧 수학적 사고력의 도야를 수학 교육의 목적으로 삼는 한, 인간이 어떻게 해서 수학적 개념이 가능한가를 파악하지 않을 수 없다. 그것이 인간이 세계를 인식하는 방식이요 사고하는 방식이기 때문이다. 수학교육의 근본적인 질문의 하나인 수학적 개념의 구성과 마음의 관계는, 칸트의 감성론과 오성론을 넘어서, 이념에 의한 순수이성의 작용, 실천이성, 판단력 일반에 대한 칸트 이론에 대한 종합적이고 심층적인 고찰이 필요하다. 특히 무조건적 통일, 자연의 합목적성과 같은 이념과 수학적 개념의 관계는 이러한 문제를 해명하는 데에 단서를 제공할 수 있을 것으로 기대된다.

2-3 정승달(제주대학교) : Variation formulas for transversally harmonic maps

In this talk, we give the definition of the transversally harmonic map between foliated Riemannian manifolds and study the variation formulas for the transversal energy. And we prove that transversally harmonic map is a critical point of the transversal energy.

2-4 김영희(*) (한남대학교), 정호용(한남대학교), 유천성(한남대학교) : A study of generalized Euler polynomials of the second kind

In this paper, our aim is finding the term of generalized Euler polynomials. We also obtain some identities involving the Genocchi numbers, the Bernoulli numbers, the Euler numbers and the Stirling numbers.

2-5 김대산(*)(서강대학교), 김태균(광운대학교) : Some identities arising from Sheffer sequences for the powers of Sheffer pairs under umbral composition

In this paper, we study some properties of Sheffer sequences for the powers of Sheffer pairs under umbral composition. From our properties we derive new and interesting identities of Sheffer sequences of special polynomials for the powers of Sheffer pairs under umbral composition.

2010 Mathematics Subject Classification. 05A19, 05A40.
Keywords and phrases. Sheffer Sequence, Sheffer Pair, Umbral Composition.

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[3] T. Kim, *Symmetry p-adic invariant integral on* $\mathbf{Z}_p$ *for Bernoulli and Euler polynomials*, J. Difference Equ. Appl. 14(2008), no. 12, 1267–1277.
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[5] E. Sen, *Theorems on Apostol–Euler polynomials of higher order arising from Euler basis*, Adv. Stud. Contemp. Math. 23 (2013), no. 2.

2-6 강정욱(*) (한남대학교), 유천성(한남대학교) : A research on the recursive value and general terms of the analogue Euler zeta function on the positive integers

Our aim is finding the general term of the analogue Euler zeta function in the positive integers by using Fourier series. We also figure out the generalized coefficients of Fourier series and research some interesting relation in the integers.

2000 Mathematics Subject Classification :11B68, 11S40, 11S80

Key words: Fourier series, cosine series, sine series, analogue Euler zeta function

2-7 최은미(한남대학교), 조지인(*) (한남대학교) : Lucas numbers and modified continued fraction

The purpose of this work is to study Lucas sequence. As a generalization of the golden ratio of n-th term by $(n-1)$-th term, we investigate the ratio of n-th term by any k-th term and express the ratio in terms of a little modified continued fraction.

We begin to display the Lucas sequence in five columns rectangle

$$\begin{array}{ccccc} 1 & 3 & 4 & 7 & 11 \\ 18 & 29 & 47 & 76 & 123 \\ 199 & 322 & 521 & 843 & 1364 \\ 2207 & 3571 & 5778 & 9349 & \cdots \end{array}$$

It can be seen that $11 \cdot 18 + 1 = 199, \quad 11 \cdot 47 + 4 = 521, \cdots$. That is,

$$11 \cdot L_6 + L_1 = L_{11}, \quad 11 \cdot L_8 + L_3 = L_{13}, \cdots$$

If we display the Lucas sequence in 6 columns Lucas table

$$\begin{array}{cccccc} 1 & 3 & 4 & 7 & 11 & 18 \\ 29 & 47 & 76 & 123 & 199 & 322 \\ 521 & 843 & 1364 & 2207 & 3571 & 5778 \\ 9349 & 15127 & 24476 & 39603 & 64079 & \cdots \end{array}$$

we can find relationships

$$18 \cdot 29 - 1 = 521, \quad 18 \cdot 47 - 3 = 843, \quad 18 \cdot 76 - 4 = 1364, \cdots$$

i.e., L_6 times the 2nd row and subtracting the 1st row yield the 3rd row.

These situations can be generalized. Our main theorems are as follows.

Theorem 1. $L_k \cdot L_{kt+r} + (-1)^{k-1} L_{k(t-1)+r} = L_{k(t+1)+r}$ for $1 \le t$; $1 \le r \le k$.

Theorem 2. Let $n = kt + r$ $(1 \le r \le k)$. Then the ratio L_n / L_{n-k} can be expressed by the modified continued fractions that

(1) $\langle\langle \underbrace{L_k; L_k, \cdots, L_k}_{t-1}, L_{k+r}, L_r \rangle\rangle$ if k is odd

(2) $[[\underbrace{L_k; L_k, \cdots, L_k}_{t-1}, L_{k+r}, L_r]]$ if k is even.

Theorem 3. Let $n = kt+r$ $(1 \le r \le k)$ and assume that n is sufficiently large.

(1) $\langle\ 1; 1, 1, 1, \cdots \rangle = \langle\langle \underbrace{1;\ 1, \cdots, 1}_{n-2}, 3,\ 1 \rangle\rangle$.

(2) $\langle\ 1; 1, 1, 1, \cdots \rangle^k = \begin{cases} \langle\langle \underbrace{L_k; L_k, \cdots, L_k}_{\lfloor \frac{n-1}{2} \rfloor - 1}, L_{k+r}, L_r \rangle\rangle & \text{if } k \text{ is odd} \\ [[\underbrace{L_k; L_k, \cdots, L_k}_{\lfloor \frac{n-1}{2} \rfloor - 1}, L_{k+r}, L_r]] & \text{if } k \text{ is even.} \end{cases}$

2-8 지운식(충북대학교),김영이(한양대학교),박윤정(*) (충북대학교) : Yeh Convolution of Generalized White Noise Functionals

We study the Yeh convolution of white noise functionals. We first introduce the notion of Yeh convolution of test white noise functionals and prove a dual property of the Yeh convolution. By applying the dual object of the Yeh convolution, we study the Yeh convolution of generalized white noise functionals, which is a non-trivial extension. Finally, We study relations between the Yeh convolution and Fourier-Gauss, Fourier-Mehler transform.

2-9 최병진(*) (충북대학교), 지운식(충북대학교) : Exponential Convergence Rates for Weighted Sums in Noncommutative Probability Space

We study exponential convergence rates for weighted sums of successively independent random variables in a noncommutative probability space of which the weights are in a von Neumann algebra. As applications, we study exponential convergence rates for weighted additive convolution sums of probability measures corresponding to the free additive convolution and Boolean additive convolution.

2-10 지운식(충북대학교),김영이(*)(한양대학교),박윤정 (충북대학교) : Convolutions of Generalized White Noise Functionals

We study a general definition of convolution products of test white noise functionals of which the consistency property is studied. As an application of the consistency property of the convolution product, we study an extension of the convolution to generalized white noise functionals. We also study relations between the convolution and generalized Fourier-Gauss, generalized Fourier-Mehler transforms.

제 3발표장 (사범대학 30118호실)

3-1 금영희(*)(단국대학교), 김영익 (단국대학교) : On the convergence of an iterative numerical method for multiple roots

In this paper, a new iterative method with cubic convergence is constructed and a numerical comparison is performed. We carry out some numerical examples to show their performance.

3-2 김영익 (단국대학교) : A class of optimal Jarratt-type fourth-order multiple-zero finders for nonlinear equations

This paper constructs a general class of two-point optimal Jarratt-type fourth-order methods in the following form for locating multiple roots of a nonlinear equation:

$$\begin{cases} y_n & = x_n - \gamma \cdot h, \quad \gamma \in \mathbb{R}, \\ x_{n+1} & = x_n - L(h) \times Q(v), \quad h = \frac{f(x_n)}{f'(x_n)},\ v = \frac{f'(y_n)}{f'(x_n)}, \end{cases}$$

where $L, Q \in \mathbf{C}^4(\mathbb{R})$.

Under the assumption that the root multiplicity m is known, we investigate convergence analysis and computational properties for the family. The optimality is checked in the sense of Kung-Traub's conjecture that convergence order of optimal methods without memory is 2^n for at most $n+1$ function evaluations per iteration. Explicit form of asymptotic error constants is presented to further verify the convergence order. A variety of numerical examples are included to verify the convergence behavior and the developed theory.

3-3 류근식(한남대학교) : The Translation Theorem on the Generalized Analogue of Wiener Space and its Applications

In 1944, Carmeron and Martin proved a translation theorem on the concrete Wiener space and in 1996, Chang and Chung established a translation theorem on the generalized Wiener space. In this note, We prove a translation theorem on the generalized analogue of Wiener space and show the theorems associated with our theorem.

3-4 구윤회(*)(한서대학교),임동만(청주대학교), 유춘미(충남대학교) : Boundedness in perturbed nonlinear differential systems

We consider the nonlinear nonautonomous differential system:

$$x'(t) = f(t, x(t)),\ x(t_0) = x_0, \tag{1}$$

where $f \in C(R^+ \times R^n, R^n),\ R^+ = [0, \infty)$ and R^n is the Euclidean n-space.

In this talk, we investigate bounds for solutions of the following nonlinear perturbed differential systems:

$$y' = f(t, y) + \int_{t_0}^{t} g(s, y(s))\ ds, y(t_0) = y_0.$$

3-5 이만섭(목원대학교) : Periodic points and differentiable dynamical systems

In this talks, we will be considered for the properties of periodic points in differentiable dynamical systems.

3-6 최성규(충남대학교) , Bowon Kang(*)(충남대학교), 구남집(충남대학교) : h-stability for fractional differential equations

In this talk, we recall the monotonic properties of the generalized Mittag-Leffler functions. Then we investigate the boundedness and h-stability of solutions of Caputo fractional differential equations via a fractional comparison principle and the fractional Lyapunov direct method.

3-7 이만섭(목원대학교), 박준미(*) (충남대학교) : Hyperbolicity of vector fields with shadowing - generic view point

We show that C^1-generically, the locally maximal homoclinic class of a vector field X associated to a hyperbolic periodic orbit has the shadowing property if and only if it is hyperbolic.

3-8 김학만(충남대학교), 이주리(*)(충남대학교) : Hyers–Ulam–Rassias stability of homomorphisms and derivations on normed Lie triple systems

In this talk, we prove the Hyers–Ulam–Rassias stability of homomorphisms and derivations on normed Lie triple systems for the following generalized Cauchy–Jensen additive mappings

$$r_0 f\Big(\frac{s\sum_{j=1}^{p} x_j + t\sum_{j=1}^{d} y_j}{r_0}\Big) = s\sum_{j=1}^{p} f(x_j) + t\sum_{j=1}^{d} f(y_j)$$

where r_0, s, t are nonzero real numbers.

3-9 이만섭(목원대학교), 이승희(*) (충남대학교) : Expansive homoclinic classes of C^1-generic flows

In this talk, we prove that C^1-generically, expansive homoclinic class is hyperbolic.

3-10 김학만(충남대학교), 신환용(*)(충남대학교) : n-dimensional additive functional equation and its stability

In this talk, we present the following result: Let $p \neq 1$ be a positive real number and $\theta \geq 0$ a real number. Let X be a real vector space and Y a Banach space. If a mapping $f : X \to Y$ satisfies the functional inequality

$$\Big\| \sum_{i=1}^{n} f(x_i) + \sum_{i \neq j} f(x_i - x_j) - f(\sum_{i=1}^{n} x_i) \Big\| \leq \delta + \theta \sum_{i=1}^{n} \|x_i\|^p$$

for all $x_1, x_2, ..., x_n \in X$, and $\delta = 0$ when $p > 1$, then there exists a unique additive mapping $L : X \to Y$ such that L satisfies the equation

$$\sum_{i=1}^{n} L(x_i) + \sum_{i \neq j} L(x_i - x_j) = L(\sum_{i=1}^{n} x_i),$$

and the inequality

$$\|f(x) - L(x)\| \leq \delta' + \frac{\theta'}{|2 - 2^p|} \|x\|^p, \ \forall x \in X,$$

where $\delta' = |n^2 - 3n| \|f(0)\| + \Big(\frac{2n^2 - 5n + 3}{n - 1}\Big)\delta$, and $\theta' = \Big(\frac{4n - 6}{n - 1}\Big)\theta$.

The mapping L is given by

$$L(x) = \begin{cases} \lim_{m \to \infty} 2^{-m} f(2^m x), & \text{if} \quad 0 < p < 1, \\ \lim_{m \to \infty} 2^{m} f(2^{-m} x), & \text{if} \quad p > 1 \end{cases}$$

for all $x \in X$.